

1. The position of a particular particle as a function of time is given by $\mathbf{r} = 9.60t\hat{\mathbf{i}} + 6.45\hat{\mathbf{j}} - 1.50t^2\hat{\mathbf{k}}$ m, where t is in seconds.

- [3 Pts.] (a) i. Explain which physical quantity the numbers 9.60, 6.45 and 1.50 represent.
 [3 Pts.] ii. What are the SI units for each?
 [6 Pts.] (b) Find the velocity vector of the particle at all times.
 [6 Pts.] (c) Find the acceleration vector of the particle at all times.
 [6 Pts.] (d) What are the instantaneous velocity and instantaneous acceleration at $t = 3.00$ s?
 [6 Pts.] (e) What are the average velocity and average acceleration of the particle between $t = 1.00$ s and $t = 3.00$ s?

a)

$$\begin{aligned} 9.60 &\rightarrow \text{velocity in } x \text{ direction, m/s} \\ 6.45 &\rightarrow y \text{ position, m} \\ 1.50 &\rightarrow \text{acceleration in } z \text{ direction, m/s}^2 \end{aligned}$$

b) $\vec{v} = \frac{d\vec{r}}{dt} = (9.60\hat{\mathbf{i}} - 3.00t\hat{\mathbf{k}})\text{m/s}$

c) $\vec{a} = \frac{d\vec{v}}{dt} = -3.00\hat{\mathbf{k}} \text{ m/s}^2$

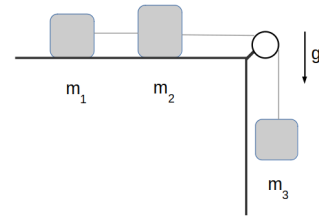
d)

$$\begin{aligned} \vec{v}(t = 3.00) &= (9.60\hat{\mathbf{i}} - 9.00\hat{\mathbf{k}})\text{m/s} \\ \vec{a}(t = 3.00) &= -3.00\hat{\mathbf{k}} \text{ m/s}^2 \end{aligned}$$

e)

$$\begin{aligned} \vec{r}(t = 3.00) &= 3 \times 9.60\hat{\mathbf{i}} + 6.45\hat{\mathbf{j}} - 9 \times 1.50\hat{\mathbf{k}} \\ \vec{r}(t = 1.00) &= 1 \times 9.60\hat{\mathbf{i}} + 6.45\hat{\mathbf{j}} - 1.50\hat{\mathbf{k}} \\ \vec{v}_{\text{avg}} &= \frac{\Delta\vec{r}}{\Delta t} = \frac{2 \times 9.60\hat{\mathbf{i}} - 8 \times 1.50\hat{\mathbf{k}}}{2.00} = (9.60\hat{\mathbf{i}} - 6.00\hat{\mathbf{k}})\text{m/s} \\ \vec{a}_{\text{avg}} &= -3.00\hat{\mathbf{k}} \text{ m/s}^2 \end{aligned}$$

2. Two blocks with masses m_1 and mass m_2 are lying on a frictionless horizontal surface. They are connected to each other and to a hanging mass, m_3 , by a cord passing over a frictionless pulley, as shown in the figure. (Ignore the masses of the cord and pulley.)



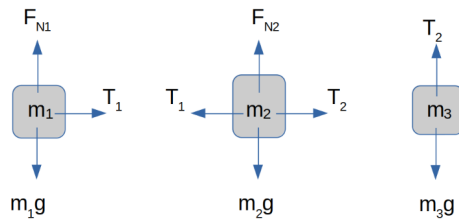
- [12 Pts.] (a) Draw a free-body diagram for each mass, showing the force of gravity on each, the tension exerted by the cords, and any normal force.

- [5 Pts.] (b) i. Write Newton's second law for each mass.

- [7 Pts.] ii. Determine a formula for the acceleration of the system in terms of m_1, m_2, m_3 and g .

- [11 Pts.] (c) Find the tensions in both cords.

a)



b)i.

$$F_{N1} = m_1g$$

$$F_{N2} = m_2g$$

$$T_1 = m_1a$$

$$T_2 - T_1 = m_2a$$

$$m_3g - T_2 = m_3a$$

ii.

Therefore $(m_1 + m_2 + m_3)a = m_3g$

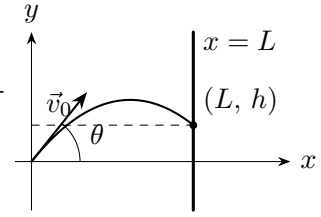
$$a = \frac{m_3}{(m_1 + m_2 + m_3)}g$$

c) $T_1 = m_1a = \frac{m_1m_3}{(m_1+m_2+m_3)}g$

and $T_2 = T_1 + m_2a = (m_1 + m_2)a = \frac{(m_1+m_2)m_3}{(m_1+m_2+m_3)}g$

3. A projectile is launched from the origin with an initial speed v_0 at an angle θ above the x-axis. A vertical wall is located at $x = L$. The gravitational acceleration is downward with magnitude g .

- [8 Pts.] (a) i. Determine the time at which the projectile hits the wall,
 [12 Pts.] ii. Find the height h of the point on the wall where the projectile hits,
 as a function of θ , v_0 , g , and L .



- [15 Pts.] (b) For $v_0 = 5$ m/s, $x = L = 2$ m, and $g = 10$ m/s², calculate the *highest* point on the wall that can be hit by varying θ .

a) i.

$$\begin{aligned} v_x &= v_0 \cos \theta, & x = v_x t \Rightarrow t_f &= \frac{L}{v_x} = \frac{L}{v_0 \cos \theta} \\ v_y &= v_0 \sin \theta \end{aligned}$$

ii. $h = y(t = t_f) = v_0 \sin \theta t_f - \frac{1}{2} g t_f^2 = L \tan \theta - \frac{1}{2} g \frac{L^2}{v_0^2 \cos^2 \theta}$

b) $\frac{dh(\theta)}{d\theta} = 0$ for maximum

$$\frac{\partial h}{\partial \theta} = L \frac{1}{\cos^2 \theta} - \frac{gL^2}{2v_0^2} \frac{2 \sin \theta}{\cos^3 \theta} = 0$$

$$\tan \theta = \frac{v_0^2}{gL} \Rightarrow \cos \theta = \frac{gL}{\sqrt{v_0^4 + g^2 L^2}}$$

$$\begin{aligned} h_{\max} &= L \frac{v_0^2}{gL} - \frac{1}{2} \frac{gL^2}{v_0^2} \frac{(v_0^4 + g^2 L^2)}{g^2 L^2} \\ &= \frac{v_0^2}{g} - \frac{v_0^2}{2g} - \frac{gL^2}{2v_0^2} \\ &= \frac{1}{2} \left[\frac{v_0^2}{g} - \frac{gL^2}{v_0^2} \right] \\ &= \frac{1}{2} \left[\frac{25}{10} - \frac{4 \times 10}{25} \right] = \frac{1}{2} \left[\frac{25 - 16}{10} \right] = 0.45 \text{ m} \end{aligned}$$